Activity Summary
Students consider the meaning of $E=mc^2$ by examining how much of different kinds of fuel would be required to make an imaginary trip to Pluto. All energy sources are compared to a hypothetical mass-to-energy propulsion system called a photon drive.

Materials for each team
- copy of “A Trip to Pluto” student handout
- copy of “Planning Your Trip” student handout
- copy of “Reaction Worksheet” student handout
- hand-held or computer calculator

Background
Albert Einstein’s genius was, in part, due to his ability to see the world as no one else could. His ideas evolved from the belief that light’s speed never changed and that nothing could exceed the speed of light. Taking this as fact, he reshaped what he knew about the universe. He came to realize that energy and matter were equivalent and that one could be transformed into the other using the speed of light squared as the conversion factor (see“The Legacy of $E=mc^2$” at www.pbs.org/nova/einstein/legacy.html for a brief explanation of the equation). Einstein’s equation was theoretical when he first thought of it, but since its proposal in 1905 it has been confirmed countless times. Scientists today continue to explore its implications.

In this activity, students explore the meaning of $E=mc^2$ by considering its effect on the fuel requirements for a trip to Pluto. Given a series of chemical reactions of fossil fuels and nuclear energy reactions, students compute how much of each fuel they would need to travel from Earth to Pluto and back. Students also consider a hypothetical energy source—a photon drive—which would convert matter to vast amounts of energy.

This activity compares chemical reactions to nuclear reactions. Students may know that mass is always conserved in chemical reactions. The same number and kinds of atoms of each of the elements exist at the beginning and end of the chemical reaction. (It is true that since light and/or heat is often absorbed or released in a reaction, some mass must have been lost or gained. But for all practical purposes this is too small to measure.)

LEARNING OBJECTIVES
Students will be able to:
- explain the meaning of $E=mc^2$.
- state that, in nuclear reactions, mass-energy is conserved.
- show that nuclear fission and fusion reactions provide many millions of times more energy than fossil fuel chemical reactions.

KEY TERMS
fossil fuel: A substance—such as coal, oil, or natural gas—that comes from the fossil remains of plants and animals. It can be burned and used as an energy source.

isotope: A form of an element that has the same number of protons but a different number of neutrons in its nucleus. Isotopes of an element have the same atomic number but different atomic weights.

nuclear fission: The splitting of a nucleus into two or more parts resulting in a large release of energy.

nuclear fusion: The combining of nuclei resulting in a large release of energy.

radioactive decay: The spontaneous disintegration of a nucleus to form a different nucleus. A large amount of energy is released during the decay.
In nuclear reactions, energy is exchanged for mass and mass for energy. Nuclei of atoms are made of protons and neutrons. When you divide a nucleus into parts, the sum of the masses of the parts is not equal to the whole (the mass of a nucleus is less than the sum of the masses of the individual protons and neutrons). This “missing” mass is accounted for by the nuclear binding energy that holds the nucleus together. The change in binding energy that is equivalent to the missing mass can be calculated using $E = mc^2$ (nuclear binding energy = $\Delta mc^2$).

Every single nuclear reaction, regardless of type or complexity, confirms the truth of $E = mc^2$. In fusion, the energy source that powers the sun and stars, light nuclei of elements such as the isotopes of hydrogen combine to form helium nuclei and release energy. This happens because the sum of the mass of the helium nucleus is less than the mass of the hydrogen nuclei fused to create it. In fission, the same is also true. The mass of the products (fission fragments and the neutrons created) is less than the mass of original reactants (uranium nucleus and neutron). Again, $E = mc^2$ predicts the energy released, which is huge. In nuclear reactions, as in chemical reactions, the total energy and mass is conserved. Thanks to Einstein, there is a way to balance the books.

The energetic fragments resulting from a nuclear fission reaction collide with surrounding matter and generate heat. It is important to stress this. Most students will simply refer to “heat” as the energy released, but that is just the end product of the process.

**Procedure**

1. Ask students what kind of fuel they would use in their car if they had to take a trip across the country. What if they had to take a much longer trip—to Pluto, for example? What type of fuel would be the best to use in a rocket ship? Discuss with students the different types of fuel available.

2. Decide whether you wish to do this activity as a class exercise or whether you want students to work in teams. Distribute student handouts and make sure students have access to calculators. If working as a class, place the table students will be working with (from the “Planning Your Trip” handout) on the board or computer.

3. You may need to review scientific notation with students. If you do this as a class exercise, you can do the calculations for students if you prefer. You may also want to review eV (electron Volt), the unit of energy used in this activity. One electron volt is equal to the energy one electron acquires when traveling across an electric potential difference of 1 volt.

4. Have students first read the “A Trip to Pluto” handout, and then “Planning Your Trip” and “Reaction Worksheet” handouts. After students have read all the handouts, help them do the calculations for each fuel source. Once students have completed their calculations, have them answer the questions on their “Planning Your Trip” handout.
To conclude the activity, examine the table with students and review the answers to student handout questions. Ask students what surprised them the most about their results. Students may ask why all spacecraft don’t use fission or fusion engines. Mention that fission reactors are very difficult to scale up because reactors need moderating rods, water to absorb energy, heavy shielding to absorb harmful radiation, etc. Even though reactor-grade fuel is less than 2 percent pure, it would take a lot of mass to shield the astronauts from the radiation that is emitted. Fusion reactions require very high temperatures and pressures to initiate the reaction and are currently only in experimental stages of development.

To illustrate the differences between the final results more clearly, ask students how they might calculate how many gallons of gasoline (instead of grams) are equivalent to the energy derived from 1.5 grams of pure matter conversion—a little more than the mass of an average ladybug. (Students just calculated that 2.3 x 10^9 grams of gasoline are needed to supply the 8 x 10^32 electron volts required for the trip to Pluto.) To convert grams of gasoline to gallons, students need to find how many grams there are in a gallon of gasoline and then convert. (A gallon of gasoline contains 2,720 grams.) The conversion is: 2.3 x 10^9 grams of gasoline x 1 gallon/2,720 grams = 8.5 x 10^1 gallons (850,000 gallons). That is a good indicator of what scientists mean when they claim Einstein unlocked the power of the atom. Converting the other fuel quantities from grams to pounds or tons may help students grasp the vast differences in amounts of fuel needed. (For example, you would need 21 million pounds of wood to complete the trip!)

As an extension, have students calculate the weight of other supplies they would need for the trip (such as food and water). Students can also calculate how big a spaceship would be required for fuel storage and living quarters, and other necessities. The energy needed to lift a kilogram of mass from Earth’s surface and escape the planet’s gravitational field is 6.3 x 10^7 J/kg or 3.9 x 10^6 eV/kg.

The figure used in this activity as the energy needed to make a round trip to Pluto—8 x 10^32 eV—attempts to consider the escape velocity, deceleration, and acceleration needed to make the trip. It does not take into account other, more complex aspects (such as variability in speed and trajectory) that occur during actual space travel. There are many options for calculating trip energy to Pluto and back. The trip energy used for this activity is based on needing an estimated 955 million joules per kilogram of mass, or 6.0 x 10^27 electron volts per kilogram, to complete the journey. Assuming a spacecraft with a mass of 135,000 kilograms brings the total energy for the trip to 8 x 10^32 eV.
ACTIVITY ANSWER

The mass of each wood or fossil fuel molecule was obtained by finding the mass of one mole in grams, then dividing by $6.02 \times 10^{23}$ molecules per mole. In nuclear reactions, the mass is calculated by summing the number of protons and neutrons reacted (measured in atomic mass units), then multiplying by $1.7 \times 10^{-24}$ grams per amu.

<table>
<thead>
<tr>
<th>Fuel Type</th>
<th>Mass (g) per Molecule</th>
<th>Energy Released per Molecule (eV)</th>
<th># Reactants Needed for Round Trip</th>
<th>Total Mass (g) of Fuel Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>wood</td>
<td>$3.0 \times 10^{-22}$</td>
<td>25</td>
<td>$3.2 \times 10^{31}$</td>
<td>$9.6 \times 10^{9}$</td>
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<td>66</td>
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<tr>
<td>fission</td>
<td>$4.0 \times 10^{-24}$</td>
<td>$230 \times 10^{6}$</td>
<td>$3.5 \times 10^{24}$</td>
<td>$1400$</td>
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<tr>
<td>fusion</td>
<td>$1.7 \times 10^{-23}$</td>
<td>$20 \times 10^{6}$</td>
<td>$4.0 \times 10^{25}$</td>
<td>$680$</td>
</tr>
<tr>
<td>photon drive</td>
<td>$3.4 \times 10^{-24}$</td>
<td>$1877 \times 10^{6}$</td>
<td>$4.3 \times 10^{23}$</td>
<td>$1.5$</td>
</tr>
</tbody>
</table>

Student Handout Questions

1. What do all the reactants of wood and fossil fuels have in common? The reactants of wood and fossil fuels are all carbon-based. Also, each reaction requires oxygen to begin burning.

2. Compare the products of wood and fossil fuel reactions with the products of nuclear reactions. How are they the same? How are they different? The products of wood and fossil fuel reactions are largely the same—water, carbon dioxide, and soot (except for natural gas, which burns cleaner than the others). Nuclear fission has radioactive isotopes as a product (students cannot tell this from the equation) and different isotopes can occur. Also, neutrons are often emitted in both fission and fusion reactions.

3. Compared to pure uranium fission, how many times more wood would you have to burn to make the trip to Pluto? How many times more wood compared to a photon drive engine? Dividing the amount of wood by the amount of uranium, you need $6.9 \times 10^{9}$ million times more wood than uranium, and $6.4 \times 10^{9}$ billion times more wood than photon drive fuel.

4. If Pluto is $5.9 \times 10^9$ kilometers from Earth, how long will it take you, in years, to make the trip to Pluto and return home? (Assume a straight line, a constant velocity with no deceleration or acceleration, and a speed of 12.0 kilometers per second.) Calculation:

   \[
   d = \nu \times t, \quad \text{where } d = \text{distance}, \ \nu = \text{velocity}, \ \text{and } t = \text{time}
   \]

   \[
   t = \frac{d}{\nu} = \frac{5.9 \times 10^9 \ \text{km} \times 2}{12.0 \ \text{km/sec} \times 3600 \ \text{sec/hr} \times 24 \ \text{hr/d} \times 365 \ \text{d/yr}} = 31.2 \text{ years}
   \]
When Albert Einstein wrote a three-page paper in 1905 outlining his theory that $E=mc^2$, there were no references to anyone else's work or ideas. Einstein could reason in words and mathematics why energy and mass are simply two forms of the same thing, but he could not confirm it. Confirmation in science comes in the form of the verification and validation of a testable statement—not just once but thousands of times by different groups of scientists. $E=mc^2$ was dramatically confirmed following the 1938 generation of fission by German scientists Otto Hahn and Fritz Strassmann, and the realization by Lise Meitner and Otto Robert Frisch that mass was being converted to energy. Since then, the fact that energy can be converted to mass, and that mass can be converted to energy, has been shown countless times. In fact, every single nuclear reaction is testimony to Einstein’s theory.

You may know that, in chemical reactions, mass is always conserved. The atoms that make up the molecules on one side of a reaction (the reactants) recombine to form different molecules on the other side of a reaction (the products). The outermost electrons of atoms interact to form these new molecules. Energy is absorbed or given off in a chemical reaction.

In nuclear reactions, mass is never conserved—some mass is exchanged for energy and energy for mass. Nuclear reactions take place in an atom’s nucleus. In a spontaneous nuclear reaction, such as radioactive decay, mass is “lost” and appears as energy in the form of particles or gamma rays. However, the total mass and energy is always conserved. One simple method of accounting used by nuclear scientists and elementary particle physicists is to express all mass in energy units. The total energy (mass and energy) is the same before and after any nuclear reaction.

Once energetic particles are produced in a nuclear reaction, they interact with surrounding matter. As they zing along, their energy is shared through collisions with many other atoms and heat is generated. In a nuclear reactor, the rate of reaction is controlled. The energetic fission fragments heat the surrounding water, which is used to create steam and run electric generators. In a nuclear bomb, the energy released is sudden and uncontrolled. Massive destruction is caused by the tremendous heat and radiation released all at once.

In this activity, you will go on a hypothetical trip to the planet Pluto. Your task is to examine the possible fuel sources for your rocket engine and compute how much of each fuel you will need for the trip. You will compare wood and fossil fuels (chemical reactions) with fission and fusion (nuclear reactions). You will also consider the fuel efficiency of a hypothetical photon drive. Use your “Planning Your Trip” handout to get started.
Procedure

1. Read your “Reaction Worksheet” handout carefully.
2. For your trip to Pluto, you will be riding in a spacecraft that has a mass of 135,000 kilograms. Most of the mass is in the rocket boosters, which are needed for taking off and escaping Earth’s gravitational field. Your task is to travel to Pluto, land on its surface, take samples of the surface ice and rocks, and then return to Earth. To accomplish this trip, you will require a lot of energy—a total of $8 \times 10^{32}$ electron Volts (eV)!
   
   Your cruising velocity will be 12.0 kilometers per second (a speedy 27,000 miles per hour!).
3. Refer to the equations on your “Reaction Worksheet” handout to obtain the amount of energy released per molecule burned (or reaction occurred) for each of your fuels. Write these in the tables below. To find the number of reactants (or reactions) you need for your round trip, divide your total round trip energy by the energy released per molecule (or reaction) for each fuel type/process listed. Record your results.

Sample calculation for wood

$$8 \times 10^{32} \text{ eV} \times \frac{1 \text{ molecule of wood}}{25 \text{ eV}} = 3.2 \times 10^{31} \text{ wood molecules} \quad (\# \text{ of reactants needed for round trip})$$

Questions

Write your answers on a separate sheet of paper.

1. What do all the reactants of wood and fossil fuels have in common?
2. Compare the products of wood and fossil fuel reactions with the products of nuclear reactions. How are they the same? How are they different?
3. Compared to pure uranium fission, how many times more wood would you have to burn to make the trip to Pluto? How many times more wood compared to a photon drive engine?
4. If Pluto is $5.9 \times 10^9$ kilometers from Earth, how long will it take you, in years, to make the trip to Pluto and return home? (Assume a straight line, a constant velocity with no deceleration or acceleration, and a speed of 12.0 kilometers per second.)

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Wood and Fossil Fuel Reactions
Wood and fossil fuel reactions require oxygen for combustion to unleash the chemical energy stored in the fuel. The energy given off by these reactions, which is expressed in electron volts, is the energy released per molecule of fuel burned. An electron volt is an exceedingly small unit equal to \(1.6 \times 10^{-19}\) joule. (One joule is the amount of energy gained by a golf ball if it falls 2.2 meters.) The subscripts refer to the number of atoms of each element in the compounds. These equations are not balanced.

- **Wood (cellulose)**
  \[ [C_{n}H_{m}O_{p}]_{q} \rightarrow CO_{2} + H_{2}O + \text{soot} + E \ (25 \text{ eV}) \]
  
- **Coal**
  \[ C + \text{some S} + O_{2} \rightarrow SO_{2} + CO_{2} + \text{soot} + E \ (2.5 \text{ eV}) \]

- **Natural Gas (methane)**
  \[ CH_{4} + O_{2} \rightarrow CO_{2} + H_{2}O + E \ (9.2 \text{ eV}) \]

- **Gasoline (octane)**
  \[ C_{8}H_{18} + O_{2} \rightarrow CO_{2} + H_{2}O + \text{soot} + E \ (66 \text{ eV}) \]

Nuclear Fuel Reactions
Nuclear reactions require mechanisms other than combustion with oxygen to unlock their energy. For fission reactions, an unstable nucleus must capture a neutron before it splits into pieces and releases its energy. In nuclear fusion, particles need to be squeezed very tightly together with high temperature and pressure. Fusion occurs naturally in the core of the sun because the temperature there is 10 million degrees F and the pressure is incredibly high! In all nuclear reactions, it is the nucleus that changes in some way, one atom at a time. The numbers you see represent the total number of protons and neutrons in each nucleus or particle.

- **Nuclear Fission**
  \[ ^{235}_{92}U + ^{0}_{0}n \rightarrow ^{92}_{56}Kr + ^{14}_{56}Ba + 3^{0}_{0}n \]
  with total \(E^\dagger \ (230 \times 10^{6} \text{ eV}) \)

- **Nuclear Fusion**
  \[ ^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n \]
  with total \(E^\dagger \ (20 \times 10^{6} \text{ eV}) \)

Photon Drive Reaction
Some scientists have considered the possibility that a photon drive, which would operate according to the principles of \(E=mc^2\), could someday power spaceships. In particle accelerators on Earth, electrons have been observed colliding with anti-electrons (called positrons). When they collide, the particles disappear and energy in the form of gamma rays appears with energy equal to the mass of both particles, as predicted by \(E=mc^2\). In a hypothetical photon drive, protons and anti-protons would be used (being almost 2,000 times more massive than electrons, they would provide more energy). If a stream of protons could be made to collide with a stream of anti-protons, tremendous amounts of energy would be emitted, and gamma rays would shoot out the back of the rocket. The reaction would look like:

\[ ^{1}_{1}p + ^{1}_{1}\bar{p} \rightarrow \text{gamma rays with total } E^\dagger \ (1877 \times 10^{6} \text{ eV}) \]